

# Decay Property of Axial-Vector Tetra-Quark Mesons

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Decay properties of hidden-charm and exotic axial-vector tetra-quark mesons are studied. It is seen that estimated width of hidden-charm odd  $\mathcal{C}$  iso-triplet partners of  $X(3872)$  as tetra-quark axial-vector mesons, although still crude, is compatible with the measured ones of recently observed  $Z_c^{\pm,0}(3900)$ . It is pointed out that  $\hat{\delta}^c(3200)$  (an indication of  $\eta\pi^0$  peak around 3.2 GeV observed in  $\gamma\gamma$  collision) gives a clue to select a realistic model of multi-quark mesons.

Recently new resonances  $Z_c^{\pm}(3900)$  have been discovered in  $\pi^{\pm}J/\psi$  channels of  $e^+e^- \rightarrow Y(4260) \rightarrow \pi^+\pi^-J/\psi$  [1], and just after the observation, their existence has been confirmed and they are named as  $Z^{\pm}(3895)$  [2]; then, not only  $Z_c^{\pm}(3900)$  but also  $Z_c^0(3900)$  have been observed in data on  $e^+e^- \rightarrow \psi(4160) \rightarrow \pi^+\pi^-J/\psi$  and  $\pi^0\pi^0J/\psi$  from CLEO-c [3]. Their measured masses and widths are as follows;  $m(Z_c^{\pm}(3900)) = (3899.0 \pm 3.6 \pm 4.9)$  MeV and  $\Gamma(Z_c^{\pm}(3900)) = (46 \pm 10 \pm 20)$  MeV in [1],  $m(Z^{\pm}(3895)) = (3894.5 \pm 6.6 \pm 4.5)$  MeV and  $\Gamma(Z^{\pm}(3895)) = (63 \pm 24 \pm 26)$  MeV in [2], and  $m(Z_c^0(3900)) = (3885 \pm 5)$  MeV,  $m(Z_c^0(3900)) = (3907 \pm 12)$  MeV and  $\Gamma(Z_c^0(3900)) = (34 \pm 13)$  MeV,  $\Gamma(Z_c^0(3900)) = (34 \pm 29)$  MeV in [3]. ( $J/\psi$  is written as  $\psi$  hereafter.) They are considered as hidden-charm iso-triplet partners of  $X(3872)$  with an opposite charge-conjugation ( $\mathcal{C}$ ) property [4] which have been predicted in our earlier studies [5–7]. In addition, an indication of  $\eta\pi^0$  peak around 3.2 GeV has been observed in  $\gamma\gamma$  collision [8]. It is considered as the neutral component of hidden-charm iso-triplet scalar mesons which are now named as  $\hat{\delta}^{c\pm,0}(3200)$ . These mesons as well as the well-established [9]  $D_{s0}^+(2317)$  and  $X(3872)$  are considered as tetra-quark mesons from their properties, as seen below. The ratio of decay rates  $R(D_{s0}^{*+}\gamma/D_{s0}^+\pi^0) = \Gamma(D_{s0}^+(2317) \rightarrow D_{s0}^{*+}\gamma)/\Gamma(D_{s0}^+(2317) \rightarrow D_{s0}^+\pi^0)$  is experimentally constrained as [9]  $R(D_{s0}^{*+}\gamma/D_{s0}^+\pi^0)_{\text{exp}} < 0.059$ . From this fact, it is natural to consider that  $D_{s0}^+(2317)$  is a member of iso-triplet states, because of the well-known hierarchy of hadron interactions [10],  $|\text{isospin conserving strong int.} (\sim O(1))| \gg |\text{radiative int.} (\sim O(\sqrt{\alpha}))| \gg |\text{isospin non-conserving hadronic int.} (\sim O(\alpha))|$ , where  $\alpha$  is the fine structure constant. Such a state cannot be a  $\{c\bar{s}\}$ . Its narrow width is understood by a small overlap of color and spin wave functions (wfs.) [10], when it is assigned to a component of the scalar  $\{[cn][\bar{s}\bar{n}]\}_{I=1}$ , ( $n = u, d$ ), where the notation of tetra-quark states will be seen later. In addition, a lattice QCD study on mass of the lowest lying charm-strange scalar meson has reproduced [12] the measured one of  $D_{s0}^+(2317)$ . This implies that it is not extended but compact, so that  $D_{s0}^+(2317)$  should be a tetra-quark meson. Regarding  $X(3872)$  which was discovered in the  $\pi^+\pi^-\psi$  channel, it is known that the decay  $X(3872) \rightarrow \pi^+\pi^-\psi$  proceeds through the intermediate  $\rho^0\psi$  state [13, 14]. Nevertheless,  $X(3872)$  is considered to be an iso-singlet state, because its charged partners have not been observed [15]. In addition, it decays into  $\gamma\psi$  [16, 17], and hence its  $\mathcal{C}$ -parity is even. Concerning with its spin and parity ( $J^P$ ), they have been determined [18] to be  $J^P = 1^+$ . Thus, quantum numbers of  $X(3872)$  are the same as those of the charmonia  $\chi_{c1}(1P)$ ,  $\chi_{c1}(2P)$ ,  $\dots$  with  $J^{PC} = 1^{++}$ . If it were a charmonium, however, the ratio of decay rates  $R(\gamma\psi/\pi^+\pi^-\psi) = \Gamma(X(3872) \rightarrow \gamma\psi)/\Gamma(X(3872) \rightarrow \pi^+\pi^-\psi)$  would be much larger than unity [6], i.e.,  $R(\gamma\psi/\pi^+\pi^-\psi)_{c\bar{c}} \gg 1$ , for the decay in the denominator would be suppressed because of the isospin non-conservation and the OZI-rule [19]. This result contradicts with the measurements,

$$R(\gamma\psi/\pi^+\pi^-\psi)_{\text{exp}} = 0.22 \pm 0.09 \text{ (Belle [16]) and } 0.33 \pm 0.12 \text{ (Babar [17])} \quad (1)$$

and hence, it should be a multi-quark state. Here, an argument that the measured cross section for prompt  $X(3872)$  production [20] favors a compact object like a tetra-quark state over an extended one like a loosely bound meson-meson molecule [21] should be noted. If it is the case,  $X(3872)$  would be a tetra-quark meson. Concerning with  $Z_c(3900)$  (or  $Z(3895)$ ), they cannot be charmonia and their neutral component  $Z_c^0(3900)$  (or  $Z^0(3895)$ ) has an odd  $\mathcal{C}$ -parity, so that they are interpreted as iso-triplet tetra-quark partners of  $X(3872)$  with an opposite  $\mathcal{C}$ -property. As for  $\hat{\delta}^{c0}(3200)$ , it will be a charmonium-like state because its mass is between those of  $\psi$  and  $\chi_{c0}$ , while its isospin is  $I = 1$ . This means that it cannot be a charmonium. The simplest way to understand it is to assign it to a tetra-quark state [22]  $\hat{\delta}^{c0} \sim \{[cn][\bar{c}\bar{n}]\}_{I=1}^0$ . In this case, its mass is estimated to be  $m(\hat{\delta}^c) \simeq 3.3$  GeV, by using the same quark counting with  $\Delta m_{cs} = m_c - m_s \simeq 1.0$  GeV and  $\Delta m_{sn} = m_s - m_n \simeq 0.1$  GeV as in [23] and taking  $m(D_{s0}^+(2317))$  as the input data. The result is close to the measured  $m(\hat{\delta}^c(3200)) \simeq 3.2$  GeV. In contrast, the diquark-antidiquark model [24] and the unitarized chiral one [25] predicted that the mass of hidden-charm scalar (which was named as  $X_0$ ) is  $m(X_0) \simeq 3.7$  GeV which is much higher than  $m(\hat{\delta}^c(3200)) \simeq 3.2$  GeV. Therefore,  $\hat{\delta}^{c0}(3200)$  will provide an important clue to select a realistic model of multi-quark states.

We here review very briefly our tetra-quark model. Tetra-quark states can be classified into four groups

$$\{qq\bar{q}\bar{q}\} = [qq][\bar{q}\bar{q}] \oplus (qq)(\bar{q}\bar{q}) \oplus \{[q\bar{q}](\bar{q}q) \oplus (q\bar{q})[\bar{q}q]\}, \quad (2)$$

in accordance with difference of symmetry property of their flavor wfs., where parentheses and square brackets denote symmetry and anti-symmetry, respectively, of flavor wfs. under exchange of flavors between them [26]. However, the second term  $(qq)(\bar{q}\bar{q})$  on the right-hand-side (r.h.s.) of Eq. (2) is not considered in this note, because no signal of scalar  $(K\pi)_{I=3/2}$  meson which can arise from  $(qq)(\bar{q}\bar{q})$  in the light flavor version has been observed in a sufficiently wide energy region  $\lesssim 1.8$  GeV [27]. Each term on the r.h.s. of Eq. (2) is again classified into two groups [26] with  $\bar{\mathbf{3}}_c \times \mathbf{3}_c$  and  $\mathbf{6}_c \times \bar{\mathbf{6}}_c$  of the color  $SU_c(3)$ . Here, the former is taken as the lower lying state in heavy mesons [23]. Regarding their  $J^P$ , the first term and the last two on the r.h.s. of Eq. (2) have  $J^P = 0^+$  and  $1^+$ , respectively, in the flavor symmetry limit, because  $[qq]$  and  $(qq)$  have  $J^P = 0^+$  and  $1^+$ , respectively, in the same limit. However, the flavor symmetry is broken in the real world, so that  $[qq]$  and  $(qq)$  can have both of  $J^P = 0^+$  and  $1^+$  in general, and hence each term on the r.h.s. of Eq. (2) can have all of  $J^P = 0^+$ ,  $1^+$  and  $2^+$ . Along with this line, the diquark-antidiquark model [24] in which a badly broken flavor symmetry (and even a badly broken isospin symmetry) are assumed assigns axial-vector mesons to  $[qq][\bar{q}\bar{q}]$  states, and hence,  $X(3872)$  to an element of  $[cn][\bar{c}\bar{n}]$ . As the result, it predicts  $m(X_0) \simeq 3.7$  GeV as the mass of hidden-charm scalar which is much higher than  $m(\hat{\delta}^c(3200))$ , and therefore, the diquark-antidiquark model fails to understand it. It is because  $X(3872)$  has been assigned to an element of  $[cn][\bar{c}\bar{n}]$  with  $J^P = 1^+$  (which disappears in the flavor symmetry limit).

In contrast, we assign scalar and axial-vector tetra-quark mesons to different  $[qq][\bar{q}\bar{q}]$  and  $\{[qq](\bar{q}\bar{q}) \oplus (qq)[\bar{q}\bar{q}]\}$ , respectively, and therefore,  $D_{s0}(2317)$  and  $\hat{\delta}^c(3200)$  to  $[cn][\bar{s}\bar{n}]_{I=1}$  and  $[cn][\bar{c}\bar{n}]_{I=1}$ , respectively [22, 23]. (In this scheme, we always assume that states with the same quark contents in the same multiplet mix ideally with each other.) In the  $J^P = 1^+$  mesons,  $[qq](\bar{q}\bar{q})$  and  $(qq)[\bar{q}\bar{q}]$  are not eigenstates of  $\mathcal{C}$ -parity, so that a pair of  $[cn](\bar{c}\bar{n})$  and  $(cn)[\bar{c}\bar{n}]$ , and a hidden-strangeness pair of  $[cs](\bar{c}\bar{s})$  and  $(cs)[\bar{c}\bar{s}]$  mix with each other to form eigenstates of  $\mathcal{C}$ -parity. Thus, we have pairs of hidden-charm axial-vector meson states with opposite  $\mathcal{C}$ -parities, i.e.,  $X(\pm) \sim \{[cn](\bar{c}\bar{n}) \pm (cn)[\bar{c}\bar{n}]\}_{I=0}$ ,  $X_I(\pm) \sim \{[cn](\bar{c}\bar{n}) \pm (cn)[\bar{c}\bar{n}]\}_{I=1}$ ,  $X^s(\pm) \sim \{[cs](\bar{c}\bar{s}) \pm (cs)[\bar{c}\bar{s}]\}$ , where  $\pm$  denotes the  $\mathcal{C}$ -parities of their neutral components. (These states survives in the flavor symmetry limit in contrast to the diquark-antidiquark model.) Consequently,  $X(3872)$  is assigned to the iso-singlet  $X(+)$  with an even  $\mathcal{C}$ -parity. In this case, the measured ratio Eq. (1) can be easily reproduced, by assuming that the isospin non-conserving decay in the denominator proceeds through the  $\omega\rho^0$  mixing [6] which plays important roles in the observed  $\omega \rightarrow \pi\pi$  decay [9] and the isospin non-conservation in nuclear forces [28]. In addition, it has been argued [7] that  $X_I(+)$  is considerably broad, when it is assumed (as an approximation) that its mass and spatial wf. are not very much different from those of  $X(+)=X(3872)$ , i.e.,  $m(X_I(+)) \simeq m(X(3872))$  and the couplings of  $X_I(+)$  to ordinary mesons (up to the Clebsch-Gordan coefficients arising from the color and spin degree of freedom) are not very far from that of  $X(+)$ . The above assumption seems to be natural, because these states belong to the same ideally-mixed  $\{[cn](\bar{c}\bar{n}) + (cn)[\bar{c}\bar{n}]\}$  multiplet. Concerning with  $X(-)$  and  $X_I(-)$ , the measured mass values of  $Z_c(3900)$  (or  $Z(3895)$ ) which is assigned to  $X_I(-)$  are close to the measured mass [9] of  $X(3872)$  which is assigned to  $X(+)$ , i.e.,

$$\frac{m(X_I(-)) - m(X(+))}{m(X(+))} = \frac{m(Z_c(3900)) \text{ (or } m(Z(3895))) - m(X(3872))}{m(X(3872))} \lesssim 1 \%. \quad (3)$$

Therefore, we again assume that the spatial wfs. of  $X(-)$  and  $X_I(-)$  are not very far from those of  $X(+)$  and  $X_I(+)$ . Under this condition, it has been intuitively expected that  $X(-)$  and  $X_I(-)$  also are considerably broad [7].

To estimate numerically the widths of  $X(-)$  and  $X_I(-) = Z_c(3900)$  (or  $Z(3895)$ ), we first estimate phenomenologically the rate for  $X(3872) \rightarrow D^0 \bar{D}^{*0}$ . We here identify  $X(3872)$  with  $X(3875)$  which was observed in the  $(D^0 \bar{D}^{*0} + c.c. \rightarrow) D^0 \bar{D}^0 \pi^0$  channel, because their measured mass values are now very close to each other. Assuming that the total rate  $\Gamma(X(3872))$  is approximately saturated as

$$\Gamma(X(3872)) \simeq \Gamma(X(3872) \rightarrow \pi^+ \pi^- \psi) + \Gamma(X(3872) \rightarrow \pi^+ \pi^- \pi^0 \psi) + \Gamma(X(3875) \rightarrow D^0 \bar{D}^{*0} + c.c.), \quad (4)$$

and taking the measured ratios of rates [29],  $[\Gamma(X(3875) \rightarrow D^0 \bar{D}^{*0} + c.c.)/\Gamma(X(3872) \rightarrow \pi^+ \pi^- \psi)]_{\text{exp}} = 9.5 \pm 3.1$  and  $R(3\pi\psi/2\pi\psi)_{\text{exp}} = 0.8 \pm 0.3$ , in addition to the measured width [30]  $\Gamma(X(3875)) = (3.9_{-1.4}^{+2.8+0.2})$  MeV, we obtain

$$\Gamma(X(3872) \rightarrow D^0 \bar{D}^{*0}) \sim (0.3 - 1.5) \text{ MeV}. \quad (5)$$

This result is consistent with an independent estimate [31],  $\Gamma(X(3872) \rightarrow D^0 \bar{D}^{*0})_{\text{Renga}} \sim 1$  MeV.

Next, we write the rate for the  $X(+)=X(3872) \rightarrow D^0 \bar{D}^{*0}$  decay as

$$\Gamma(X(+) \rightarrow D^0 \bar{D}^{*0}) = \frac{|g_{X(+)D^0 \bar{D}^{*0}}|^2}{24\pi m(X(+))^2} p_D \left\{ 2 + \frac{(m(X(+))^2 - m_{D^0}^2 + m_{\bar{D}^{*0}}^2)^2}{4m(X(+))^2 m_{\bar{D}^{*0}}^2} \right\}, \quad (6)$$

where  $g_{X(+)D^0 \bar{D}^{*0}}$  denotes the  $X(+)D^0 \bar{D}^{*0}$  coupling strength and  $p_D$  is the size of the center-of mass momentum of  $D^0$  in the final state. Under the above assumptions (as an approximation) on masses and spatial wfs. of  $X(+)=X(3872)$

Table I. Rates for OZI-rule-allowed two- and three-body decays of hidden-charm partners of  $X(3872)$  are listed, where it is assumed that the masses and spatial wave functions of  $X(\pm)$  and  $X_I(\pm)$  are nearly equal to each other. The rate  $\Gamma(X(+) \rightarrow D^0 \bar{D}^{*0}) \sim (0.3 - 1.5)$  MeV which is given in the text is taken as the input data.

Decay	Rate (MeV)	Decay	Rate (MeV)
$X_I^0(+) \rightarrow \rho^0 \psi \rightarrow \pi^+ \pi^- \psi$	$\sim 20 - 200$ ( $\ddagger$ )	$X_I^0(+) \rightarrow D^0 \bar{D}^{*0}$	$\sim 0.3 - 1.5$
$X(-) \rightarrow \eta_c \omega$	$\sim 9 - 45$	$X(-) \rightarrow \eta \psi$	$\sim 7 - 35$ (*)
$X_I^0(-) \rightarrow \pi^0 \psi$	$\sim 15 - 75$	$X_I^0(-) \rightarrow \eta_c \rho^0 \rightarrow \eta_c \pi^+ \pi^-$	$\sim 6 - 30$

( $\ddagger$ ) Ref. [7]. (\*) The  $\eta\eta'$  mixing with  $\theta_P = -20^\circ$ .

and its hidden-charm partners  $X_I(\pm)$  and  $X(-)$ , rates for OZI-rule-allowed decays of  $X_I(\pm)$  and  $X(-)$  are very crudely estimated as listed in Table I, where Eq. (5) has been taken as the input data and the broad width of  $\rho^0$  meson has been taken into account. The full width  $\Gamma(X_I(-))$  is approximately given by a sum of rates for dominant two- and three-body decays, i.e.,  $\Gamma(X_I(-)) \simeq \Gamma(X_I(-) \rightarrow \pi^0 \psi) + \Gamma(X_I(-) \rightarrow \eta_c \rho^0 \rightarrow \eta_c \pi^+ \pi^-) \sim (20 - 100)$  MeV. The full width  $\Gamma(X(-))$  also is approximately estimated in a similar way. The results imply that the widths of  $X_I(\pm)$  and  $X(-)$  are considerably broad as intuitively expected. Therefore, statistics much higher than those to observe  $X(3872)$  would be required to observe them in  $B$  decays. Fortunately, however,  $Z_c^{\pm,0}(3900)$  (or  $Z^{\pm,0}(3895)$ ) as good candidates of  $X_I(-)$  have been observed in  $e^+e^- \rightarrow Y(4260) \rightarrow \pi^+ \pi^- \psi$ , and  $e^+e^- \rightarrow \psi(4160) \rightarrow \pi^+ \pi^- \psi$  and  $\pi^0 \pi^0 \psi$ . It should be noted that our estimate of  $\Gamma(X_I(-))$  is consistent with the measured values of their widths.

Our tetra-quark interpretation of  $D_{s0}^+(2317)$ ,  $\hat{\delta}^c(3200)$  and  $X(3872)$  seems to be favored by experiments. In addition, the measured mass and width of  $Z_c^{\pm,0}(3900)$  (or  $Z^{\pm,0}(3895)$ ) are consistent with our predictions. However, to establish our tetra-quark interpretation, observation of states which have been predicted in our model would be needed. In this sense, confirmation of existence of the iso-singlet partner  $\{[cn][\bar{s}\bar{n}]\}_{I=0}^+$  of  $D_{s0}^+(2317) \sim \{[cn][\bar{s}\bar{n}]\}_{I=1}^+$  whose indication has already been observed in the  $D_s^{*+}\gamma$  channel from  $B$  decays [32] would take priority. (However, it should be noted that production of such a state in  $e^+e^-$  annihilation is suppressed. This can be understood by considering their production in a framework of minimal  $q\bar{q}$  pair creation [33].) In addition, our tetra-quark model has predicted [23, 34] existence of scalar and axial-vector mesons with exotic quantum numbers. Therefore, their observation is one of important options to establish our tetra-quark interpretation.  $D_{s0}^0(2317)$  and  $D_{s0}^{*+}(2317)$  are examples of such states, where  $D_{s0}^{0,+}(2317) \sim [cn][\bar{c}\bar{n}]_{I=1}^{0,+}$  in the present scheme [23]. Although they have not been observed in inclusive  $e^+e^-$  annihilation [35], it does not necessarily imply their non-existence but it implies that their production is *suppressed* in the inclusive  $e^+e^-$  annihilation [33]. On the other hand, branching fractions for their productions in  $B$  decays have been estimated as [33]  $Br(B_u^+ \rightarrow D^{(*)-} D_{s0}^{*+}(2317)) \sim Br(B_d^0 \rightarrow \bar{D}^{(*)0} D_{s0}^0(2317)) \sim (10^{-4} - 10^{-3})$ . The results seem to be large enough to observe  $D_{s0}^0(2317)$  and  $D_{s0}^{*+}(2317)$ . Therefore, their observation in  $B$  decays is awaited.

Detection of axial-vector mesons  $H_{Acc}^+ \sim (cc)[\bar{u}\bar{d}]$  and  $K_{Acc}^{+,+} \sim (cc)[\bar{n}\bar{s}]$  with  $C = 2$  is another option. Their masses can be very crudely estimated as  $m_{H_{Acc}} \simeq 3.87$  GeV and  $m_{K_{Acc}} \simeq 3.97$  GeV by using the same quark counting as before, where  $m_{(cc)[\bar{u}\bar{d}]} \simeq m_{(cn)[\bar{c}\bar{n}]} \simeq m_{[cn](\bar{c}\bar{n})} \simeq m_{X(3872)}$  has been assumed. These results are very close to thresholds of possible OZI-rule-allowed two- and three-body decays of these mesons. (Deviations between their estimated masses and thresholds under consideration are probably much smaller than the uncertainties involved in their estimated mass values.) These meson states might correspond to a part of  $T_{cc}$ 's by Lee and Yasui [36], although their mass values estimated in these two different models are not necessarily agree with each other. If  $T_{cc}$ 's are stable against their OZI-rule-allowed strong decays, as discussed in [36], they should be narrow and could be observed as sharp peaks in  $DD_{(s)}\gamma$  channels in  $B_c$  decays and in inclusive  $e^+e^-$  annihilation if their production rate is sufficiently high. On the other hand, in our case, it is very delicate whether their OZI-rule-allowed strong decays are kinematically allowed or not, because the estimated masses of these exotic mesons, although they include large uncertainties as discussed above, are very close to corresponding thresholds of the OZI-rule-allowed decays. (Even though their true masses are a little bit higher than the thresholds of these decays, they would be narrow because of their small phase space volume. However, we here do not estimate their widths, because it is difficult to get a definite result under the present condition.) Regarding  $K_{Acc}$ 's, they will be observed as narrow resonances in  $DD_s^+\gamma$  (and  $DD_s^+\pi$  if kinematically allowed) channels in  $B_c$  decays, if branching fractions for  $B_c$  decays producing them, which have been estimated very crudely as [34]  $Br(B_c^+ \rightarrow \{\bar{D}^{(*)} K_{Acc}\}^+) \sim (10^{-4} - 10^{-3})$ , are large enough. In contrast, observation of  $H_{Acc}^+$  in  $B_c$  decays would be not very easy, because its production in  $B_c$  decays is CKM suppressed [34]. Here, it should be noted that observation of double-charm mesons will exclude the diquark-antidiquark model.

Observation of exotic  $C = -S = 1$  meson states is an additional option. However, the scalar  $\hat{E}^0 \sim [cs][\bar{u}\bar{d}]$  decays only through weak interactions [23], so that its observation might not be easy. Regarding axial-vector tetra-quark mesons  $E_{A(cs)}^0 \sim (cs)[\bar{u}\bar{d}]$  and  $E_{A[cs]}^{+,0,-} \sim [cs](\bar{n}\bar{n})$ , their masses have been very crudely estimated [34] to be 2.97 GeV by using the same quark counting as the above. The resulting mass values are sufficiently higher than thresholds

Table II. Rates for two-body decays of  $E_{A(cs)}^0$  and  $E_{A[cs]}^+$  are listed, where the rate  $\Gamma(X(+) \rightarrow D^0 \bar{D}^{*0}) \sim (0.3 - 1.5)$  MeV which is given in the text is taken as the input data.

Decay	Rate (MeV)	Decay	Rate (MeV)
$E_{A(cs)}^0 \rightarrow \bar{K}^0 D^{*0}$	$\sim 6 - 30$	$E_{A(cs)}^0 \rightarrow K^- D^{*+}$	$\sim 6 - 30$
$E_{A(cs)}^0 \rightarrow \bar{K}^{*0} D^0$	$\sim 5 - 25$	$E_{A(cs)}^0 \rightarrow K^{*-} D^+$	$\sim 5 - 25$
$E_{A[cs]}^+ \rightarrow \bar{K}^0 D^{*+}$	$\sim 12 - 65$	$E_{A[cs]}^+ \rightarrow \bar{K}^{*0} D^+$	$\sim 10 - 50$

of their possible OZI-rule-allowed two-body decays. Therefore, it can be expected that they are considerably broad, because of large phase space volume. To study numerically their decay rates, we here decompose  $E_{A(cs)}^0$  into a sum of products of  $\{q\bar{q}\}$  pairs, and then replace the color singlet  $\{q\bar{q}\}$  pairs by the ordinary mesons, as in [7]. The result is

$$E_{A(cs)}^0 = \frac{1}{4\sqrt{3}} \left\{ D^{*+} K^- + D^+ K^{*-} - D^{*0} \bar{K}^0 - D^0 \bar{K}^{*0} + \bar{K}^{*0} D^0 + \bar{K}^0 D^{*0} - K^{*-} D^+ - K^- D^{*+} \right\} + \dots, \quad (7)$$

where  $\dots$  denotes a sum of products of color octet  $\{q\bar{q}\}_{8_c}$  pairs.  $E_{A[cs]}^{-,0,+}$  also can be decomposed in the same way. Here, we list only the decomposition of  $E_{A[cs]}^+$  to save space,

$$E_{A[cs]}^+ = \frac{1}{2\sqrt{6}} \left\{ D^{*+} \bar{K}^0 + D^+ \bar{K}^{*0} - \bar{K}^{*0} D^+ - \bar{K}^0 D^{*+} \right\} + \dots \quad (8)$$

Rates for the  $E_{A(cs)}^0$  (or  $E_{A[cs]}^+$ )  $\rightarrow \bar{K} D^*$  [and  $D \bar{K}^*$ ] decays which are considered as their dominant ones are obtained by replacing  $X(+)$ ,  $D^0$  and  $\bar{D}^{*0}$  in Eq. (6) in terms of  $E_{A(cs)}^0$  (or  $E_{A[cs]}^+$ ),  $\bar{K}[D]$  and  $D^*[\bar{K}^*]$ , respectively. In this way, the ratios of rates  $\Gamma(E_{A(cs)}^0 \rightarrow \bar{K} D^*[D \bar{K}^*])/\Gamma(X(+) \rightarrow D^0 \bar{D}^{*0})$  are given by the ratios of coupling strengths  $|g_{E_{A(cs)}^0} \bar{K} D^*[D \bar{K}^*]/g_{X(+)} D^0 \bar{D}^{*0}|^2$ . The latter ratios are provided by the ratios of the Clebsch-Gordan coefficients in the above Eq. (7) to the ones of Eq. (10) in Ref. [7], under the assumption (as an approximation) that the spatial wf. overlaps among  $E_{A(cs)}^0$ ,  $\bar{K}[D]$  and  $D^{*0}[\bar{K}^*]$  are not very much different from that of  $X(+)$ ,  $D^0$  and  $\bar{D}^{*0}$ . In the same way, the rates for two-body decays of  $E_{A[cs]}^+$  also can be estimated as listed in Table II, where the rate in Eq.(5) has been taken as the input data.

Full widths  $\Gamma(E_{A(cs)}^0)$  and  $\Gamma(E_{A[cs]}^+)$  are approximately given by a sum of the rates for two-body decays listed in Table II, i.e.,  $\Gamma(E_{A(cs)}^0) \simeq \Gamma(E_{A[cs]}^+) \sim (20 - 100)$  MeV. From this, it is seen that they are much broader than  $X(3872)$ . On the other hand, branching fractions for  $B$  decays which produce  $E_{A(cs)}^0$  and  $E_{A[cs]}^0$  have been estimated [34] to be  $Br(\bar{B} \rightarrow \bar{D} E_{A(cs)}^0) \sim Br(\bar{B} \rightarrow \bar{D} E_{A[cs]}^0) \sim (10^{-4} - 10^{-3})$ . Production of  $E_{A[cs]}^{+,-}$  is also described by the same type of quark-line diagrams, so that the branching fractions for the  $B$  decays producing them also are expected to be of the same order of magnitude, i.e.,  $Br(\bar{B} \rightarrow \bar{D} E_{A[cs]}^{+,-}) \sim (10^{-4} - 10^{-3})$ . These results are not very much different from branching fractions of  $B$  decays producing  $D_{s0}^+(2317)$  and  $X(3872)$ . Therefore, detection of  $E_{A(cs)}^0$  and  $E_{A[cs]}^+$  in  $B$  decays will require statistics much higher than those to observe  $D_{s0}^+(2317)$  and  $X(3872)$ .

In summary, we have presented our tetra-quark interpretation of  $D_{s0}^+(2317)$ ,  $X(3872)$  and  $\hat{\delta}^c(3200)$  (the  $\eta\pi$  peak around 3.2 GeV), and have pointed out that  $\hat{\delta}^c(3200)$  can be a clue to select a realistic model of multi-quark mesons. Next, we have studied decay properties of the partners of  $X(3872)$ . However, the hidden-charm and exotic  $C = -S = 1$  partners are expected to be broad, and therefore, detection of them in  $B$  decays will require statistics much higher than the ones to observe  $X(3872)$ . Therefore, to observe them at the present experimental accuracy, some other processes should be studied. Fortunately,  $Z_c^{\pm,0}(3900)$  which are good candidates of  $X_I^{\pm,0}(-)$  in our scheme have been observed in exclusive  $e^+e^- \rightarrow \pi^+\pi^-\psi$  annihilations. Although decay property of  $X^s(\pm)$  is left as one of our future subjects, its width is intuitively expected to be narrow, because of a small phase space volume. On the other hand,  $K_{Acc}^{+,++}$  with  $C = 2$  and  $S = 1$  can be narrow. If their production rates are sufficiently high, they can be observed as narrow resonances in  $(DD_s^+\pi)$  and  $DD_s^+\gamma$  channels in inclusive  $e^+e^-$  annihilation. If not, however, they will be observed as sharp peaks in  $B_c^+ \rightarrow \{\bar{D}^{(*)} K_{Acc}\}^+ \rightarrow (\{\bar{D}^{(*)}(DD_s^+\pi)\}^+ \text{ and } \{\bar{D}^{(*)}(DD_s^+\gamma)\}^+)$  decays.

In addition, confirmation of the iso-singlet partner of  $D_{s0}^+(2317)$  in the  $D_s^{*+}\gamma$  channel and observation of  $D_{s0}^0(2317)$  and  $D_{s0}^{++}(2317)$  in  $D_s^+\pi^\pm$  channels from  $B$  decays are awaited.

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